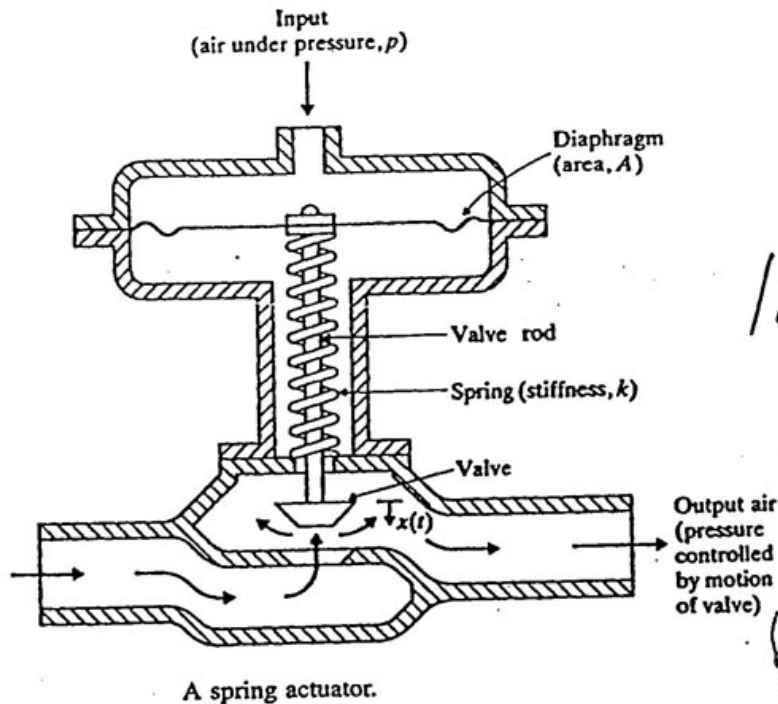


Name: _____

MEN 330
EXAM 1

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- 1) (15 pts) The spring actuator shown in the figure operates using the air pressure from a pneumatic controller (p) as input and providing an output displacement to a valve (x) proportional to the input air pressure. The diaphragm, made of a fabric base rubber, has an area A and deflects under the input air pressure against a spring of stiffness k . Find the steady-state response of the valve under a harmonically fluctuating input air pressure $p(t) = p_0 \sin \omega t$ for the following data: $p_0 = 10 \text{ psi}$, $\omega = 8 \text{ rad/s}$, $A = 100 \text{ in}^2$, $k = 400 \text{ lb/in}$, weight of spring = 15 lb, weight of valve and valve rod = 20 lb



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$$F_0 = \frac{p_0}{A} \Rightarrow F_0 = p_0 \times A = \frac{10 \text{ lb}}{\text{in}^2} \times 100 \text{ in}^2 = 1000 \text{ lb}$$

$$F(t) = F_0 \sin \omega t = 1000 \sin 8t$$

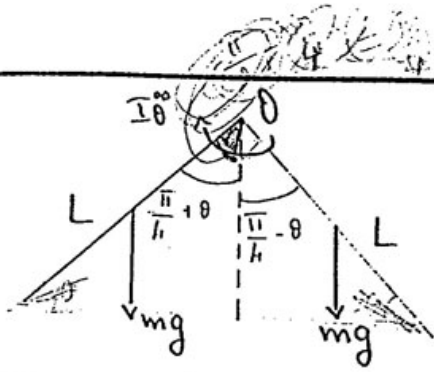
$$m_{eq} \ddot{x} + kx = F(t) = 1000 \sin 8t$$

$$\omega_n = \sqrt{\frac{k}{m_{eq}}} \quad \text{with } m_{eq} = M + \frac{m_{sp}}{3} = \frac{20}{32.2 \times 12} + \frac{15}{32.2 \times 12 \times 3} = 0.06469 \text{ kg}$$

$$\omega_n = \sqrt{\frac{400}{0.06469}} = 24.63419$$

Harmonically excited system

(Steady state)



$$\theta = \frac{\pi}{4}$$

$$(\sum M_0 = 0) \Leftrightarrow mg \frac{L}{2} \sin\left(\frac{\pi}{4} + \theta\right) - mg \frac{L}{2} \sin\left(\frac{\pi}{4} - \theta\right) - I \ddot{\theta} = 0$$

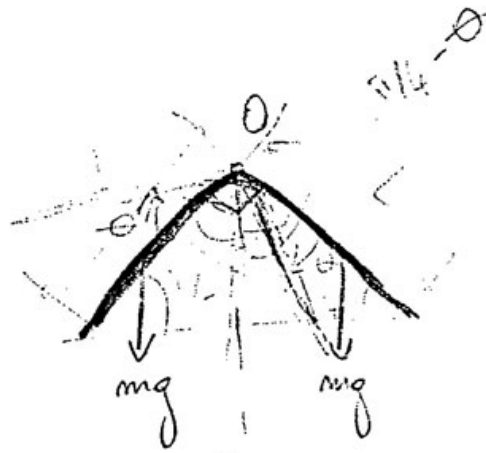
$$mg \frac{L}{2} \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \theta\right) - mg \frac{L}{2} \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \theta\right) - I \ddot{\theta} = 0$$

$$I = 2 \int r^2 dm = 2 \int_0^L r^2 \rho A dr = 2 \rho A \frac{r^3}{3} \Big|_0^L = \frac{2}{3} mL^2$$

$$\Leftrightarrow \frac{2}{3} mL^2 \ddot{\theta} + mg \frac{L}{\sqrt{2}} \theta = 0$$

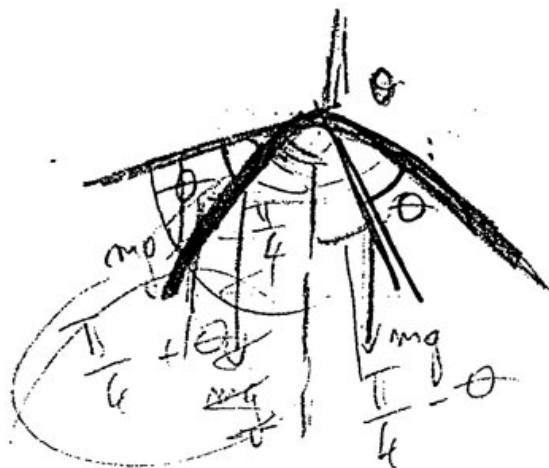
$$\frac{2}{3} L \ddot{\theta} + \frac{g}{\sqrt{2}} \theta = 0$$

$$\Leftrightarrow \omega_n = \sqrt{\frac{3g}{2\sqrt{2}L}}$$



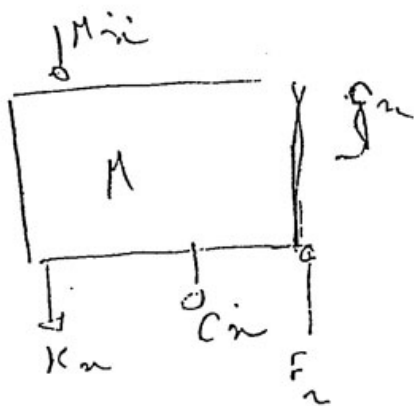
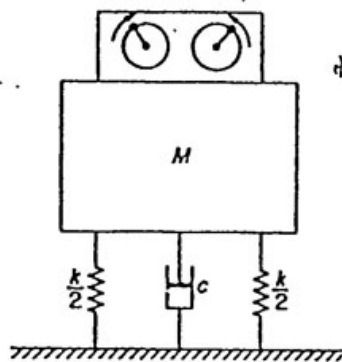
$$\frac{\pi}{2} = \theta + \frac{\pi}{4} - \theta + \alpha$$

$$\rightarrow \alpha = \frac{\pi}{4}$$



- 3) (20 pts) A counterrotating eccentric mass exciter operates on a structure having a mass of 181.4 kg. At a speed of 900 rpm, it was observed that the eccentric masses were at the top position and at that same instant, an upward deflection past the equilibrium position occurred with a magnitude of 21.6 mm. If the unbalance of each wheel of the exciter is 0.0921 kg·m, determine $\Rightarrow A_0$
- the natural frequency of the structure
 - the damping constant of the system

2 springs in // $\Rightarrow K_{eq} = K_1 + K_2$
 $\therefore K_{eq} = \frac{K}{2} + \frac{K}{2} = K$



$$\omega = \frac{900}{60} \times 2\pi = 94.247 \text{ rad/sec}$$

$$= 0.0216 \text{ m}$$

$$= 181.4 \text{ kg}$$

$$c = 0.0921$$

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2 wheels rotate with ω

$$= 2 \times 0.0921 \times (94.247)^2 \sin \omega t = 1636.15 \sin 94.247 t$$

$$M \ddot{x} + Kx + C \dot{x} = F_2$$

$$181.4 \ddot{x} + Kx + C \dot{x} = 1636.15 \sin 94.247 t$$

$$\frac{\omega_r}{\omega_n} = \frac{94.247}{\sqrt{\frac{K}{181.4}}} = \frac{1}{\sqrt{1-2\zeta^2}} \text{ (max. value } \Rightarrow r=1)$$

$$\frac{(94.247)^2 \times 181.4}{K} = 1 - 2\zeta^2 \quad \therefore K = \frac{1611284.957}{1 - 2\zeta^2}$$

$$= \frac{2 \times 0.0921 \times (94.247)^2}{181.4} \times \frac{(1 - 2\zeta^2)}{\sqrt{(1 - 2\zeta^2)^2 + (2\zeta \sqrt{1 - 2\zeta^2})^2}}$$

$$0.0216 = 1.015 \times 10^{-3} \times \frac{(1 - 2\zeta^2)}{\sqrt{(1 - 2\zeta^2)^2 + (2\zeta \sqrt{1 - 2\zeta^2})^2}}$$

$$452.48 (4\epsilon^2 - 4\epsilon^4) - 174\epsilon^4 - 4\epsilon^2$$

$$1809.93 \epsilon^2 - 1809.93 \epsilon^4 = 174\epsilon^4 - 4\epsilon^2$$

$$1813.93 \epsilon^2 - 1813.93 \epsilon^4 - 1 = 0$$

let $b = \epsilon^2$

$$-1813.93 b^2 + 1813.93 b - 1 = 0$$

$$b = 5.51 \times 10^{-4}; \quad b = 0.999$$

$$\Rightarrow \epsilon_1 = 0.02342; \quad \epsilon_2 = 0.999 \dots \text{ since } \epsilon < \frac{1}{\sqrt{2}} = 0.707$$

$$K = \frac{1611284.957}{1 - 2 \times (0.02342)^2} = 1613062.038$$

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{1613062.038}{181.34}} = 94.374 \text{ rad/sec}$$

$$\epsilon = \frac{\phi C}{2 \omega_n \times M} \Rightarrow C = \epsilon \times \omega_n \times M$$

$$C = 0.02342 \times 2 \times 94.374 \times 181.34 = 102.81$$

good